**Topic: Applications of Arithmetic and Geometric Sequences**

Time: 45 mins Marks: /45 marks

**Calculator Assumed**



**Question One: [2, 7, 2: 11 marks]**

Elsa is negotiating with her mother as to how much pocket money she will get. Elsa suggests starting with $50 in the first month and increasing this by $5 every month.

a) With this scheme, how much pocket money will Elsa receive 12 months from the start?

Elsa’s mother says that increasing the amount by 5% each month is better for Elsa in the long run.

b) i) Use the table below to show how much pocket money Elsa will receive with her scheme and her mother’s scheme for the first 5 months of the year.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Month 1 | Month 2 | Month 3 | Month 4 | Month 5 |
| Elsa’s scheme |  |  |  |  |  |
| Mother’s scheme |  |  |  |  |  |

ii) Is Elsa’s mother correct? Justify your solution mathematically.

d) If the amount of pocket money Elsa will be paid is capped $120/month, does this effect which scheme is better? Justify your solution.

**Question Two: [2, 5, 3: 10 marks]**

Scientists are tracking the size of a colony of ants. Due to deaths, ants getting lost or leaving the colony the population decreases in size by 14.5% each year. 35 new ants are born per year and they always seem to be born at the end of the year.

There were 1200 ants in the colony when scientists first started tracking this colony. Shown below is a partially completed table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Year | Ants at the start of the year | Number of deaths | Number of ants born | Population at the end of the year |
| 1 | 1200 | 174 | 35 | 1061 |
| 2 | 1061 | A | 35 | B |
| 3 | 942.155 | 136.612 | 35 | 840.543 |
|  |  |  |  |  |
| 10 | C | 68.940 | 35 | D |

a) Calculate the values of A and B in the table above.

b) Write a recursive rule to determine the population of ants at the end of each year and hence or otherwise calculate the values of C and D.

c) When will the population of the colony remain constant/stable? Explain how this is mathematically possible and state the population at this time.

**Question Three: [2, 2, 3, 2: 9 marks]**

A group of friends want to hire a bus to take them away for a camping trip. They will split the cost of the hire per person. The following graph partially models the cost per person of hiring the bus depending the number of people included.



a) Add to the graph above the cost if the bus is hired by 5 people, 10 people and 20 people.

b) Is this relationship arithmetic, geometric or neither? Justify your reasoning.

Another bus company has a completely different model for hiring out the bus. They charge a $100 bond and then $30 per person.

c) Complete the following table showing the total cost of hiring a bus from the second company.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Number of People | 1 | 2 | 3 | 20 |
| Total cost |  |  |  |  |

d) Write a recursive rule which could be used to describe the total cost of the bus hire with the second company.

**Question Four: [8, 7: 15 marks]**

The following are the first three terms of a sequence:

a) Assuming the sequence is geometric calculate the value of and state the first four terms of the sequence.

b) If the first three terms of the sequence above form the start of an arithmetic sequence, calculate the value of and state the first four terms of the sequence.

**Topic: Applications of Arithmetic and Geometric Sequences SOLUTIONS**

Time: 45 mins Marks: /45 marks

**Calculator Assumed**





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| --- | --- | --- | --- | --- | --- |
|  | Month 1 | Month 2 | Month 3 | Month 4 | Month 5 |
| Elsa’s scheme |  |  |  |  |  |
| Mother’s scheme |  |  |  |  |  |

ii) Is Elsa’s mother correct? Justify your solution mathematically.





*Elsa’s mother’s scheme is better in the long run. By the 28th month Elsa will be getting more pocket money under her mother’s scheme and it increases exponentially.*

c) If the amount of pocket money Elsa will be paid is capped $120/month, does this effect which scheme is better? Justify your solution.

*Yes, in that case, Elsa’s scheme is better because she reached $120 first under her scheme.*

**Question Two: [2, 5, 3: 10 marks]**

Scientists are tracking the size of a colony of ants. Due to deaths, ants getting lost or leaving the colony the population decreases in size by 14.5% each year. 35 new ants are born per year and they always seem to be born at the end of the year.

There were 1200 ants in the colony when scientists first started tracking this colony. Shown below is a partially completed table.

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a) Calculate the values of A and B in the table above.



b) Write a recursive rule to determine the population of ants at the end of each year and hence or otherwise calculate the values of C and D.







c) When will the population of the colony remain constant/stable? Explain how this is mathematically possible and state the population at this time.



*By the start of the 59th year the population of ants is 241. of 241 is 35. Meaning the number of ants who die is the same as the number of ants being born.*

**Question Three: [2, 2, 3, 2: 9 marks]**

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a) Add to the graph above the cost is the bus is hired by 5 people, 10 people and 20 people.

b) Is this relationship arithmetic, geometric or neither? Justify your reasoning.

*Neither, the relationship is inverse proportion. There is no constant difference or constant multiplier so it is not an arithmetic or geometric relationship.*

Another bus company has a completely different model for hiring out the bus. They charge a $100 bond and then $30 per person.

c) Complete the following table showing the total cost of hiring a bus from the second company.

|  |  |  |  |  |
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